B.Sc Semester VI

Paper - XIII

MODEL Questions

Group - A [Laplace Transform]

1> If $L\{F(t)\} = f(s)$

then prove that $L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$

for $n = 1, 2, 3 \cdots\cdots$

2> If $L\{F(t)\} = \{f(s)\}$, then

prove that $L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(x)\,dx$

provided the integral exist.

(3) Find the Laplace transform of

(i) $t \cos at$ (ii) $L\{t^2 \sin at\}$

(4) Find the Laplace transform of $\frac{\sin at}{t}$. Does the transform of $\frac{\cos at}{t}$ exist.

(5) Find inverse Laplace transform of the following

(i) $L^{-1}\left\{\frac{s}{s^2+9}\right\}$ (ii) $L^{-1}\left\{\frac{1}{s-2} + \frac{2}{s+5} + \frac{6}{s^4}\right\}$

(iii) Find $L^{-1}\left\{\frac{s-2}{(s-2)^2+5^2} + \frac{s+4}{(s+4)^2+9^2} + \frac{1}{(s+2)^2+3^2}\right\}$

(6) Change of Scale Property

If $f(s) = L\{F(t)\}$ denotes the Laplace transform of the function $F(t)$, prove that

$$L^{-1}[f(as)] = \frac{1}{a} F\left(\frac{t}{a}\right), \quad a > 0$$

(7) $L^{-1}\left[\frac{p}{(p^2+1)^2}\right] = \frac{t}{2}\sin t$, then find $L^{-1}\left[\frac{32p}{(16p^2+1)^2}\right]$.

(8) State and Prove Convolution Theorem.

or

If $L^{-1}\{f(s)\} = F(t)$ $L^{-1}\{g(s)\} = G(t)$,

then $L^{-1}\{f(s)\, g(s)\} = \int_0^t F(u)\, G(t-u)\, du$

$= F * G$

$= \int_0^t F(t-u)\, G(u)\, du$

(9) Find the inverse Laplace transform of $\frac{1}{s^2(s^2-a^2)}$ by using the Convolution theorem.

(10) Use the Convolution theorem to find $L^{-1}\left\{\frac{p^2}{(p^2+4)^2}\right\}$

(11) Using Laplace transform method, Solve $y''(t) + y(t) = t$ given that $y'(0) = 1$, $y(\pi) = 0$.

(12) Using Laplace transform Solve $y'' + y = \cos x$, where $y(0) = 0 = y'(0)$.